

2008
A.MATH



HONG KONG CERTIFICATE OF EDUCATION
MOCK EXAMINATION 2008

ADDITIONAL MATHEMATICS

Question-Answer Book

By Oscar Tam and Wister Chan

- Write your Candidate Number in the space provided on Page 1.
- Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7 and 9.
- This paper consists of **TWO** sections, Section A and Section B. Section A carries 62 marks and Section B carries 48 marks.
- Attempt **ALL** questions in Section A. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number and stick a barcode label on each sheet, and fasten them with a string **INSIDE** this book.
- Answer any **FOUR** questions in Section B. Write your answers in the CE(B) answer book.
- The Question-Answer Book and the CE(B) answer book must be handed in separately at the end of the examination.
- All working must be clearly shown.
- Unless otherwise specified, numerical answers must be **exact**.
- In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as \vec{u} in their working.
- The diagrams in this paper are not necessarily drawn to scale.

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Section A Question No.	Marks	Marks
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Section A Total		

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FORMULAS FOR REFERENCE

參考公式

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$	

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SECTION A (62 marks)

甲部 (62 分)

Answer **ALL** questions in this section and write your answers in the spaces provided in this Question-Answer Book.

本部試題均須作答，答案須寫在本試題答題簿中預留的空位內。

1. Let $f(x) = 2x\sqrt{x^2 - 1}$. Find the value of $f'(2)$.

設 $f(x) = 2x\sqrt{x^2 - 1}$ 。求 $f'(2)$ 的值。

(3 marks)

Please do not write in the margin.

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Page total

2. Figure 1 shows the graphs of $y = \cos x$ and $y = -\sin x$.

圖 1 所示 $y = \cos x$ 與 $y = -\sin x$ 的圖像。

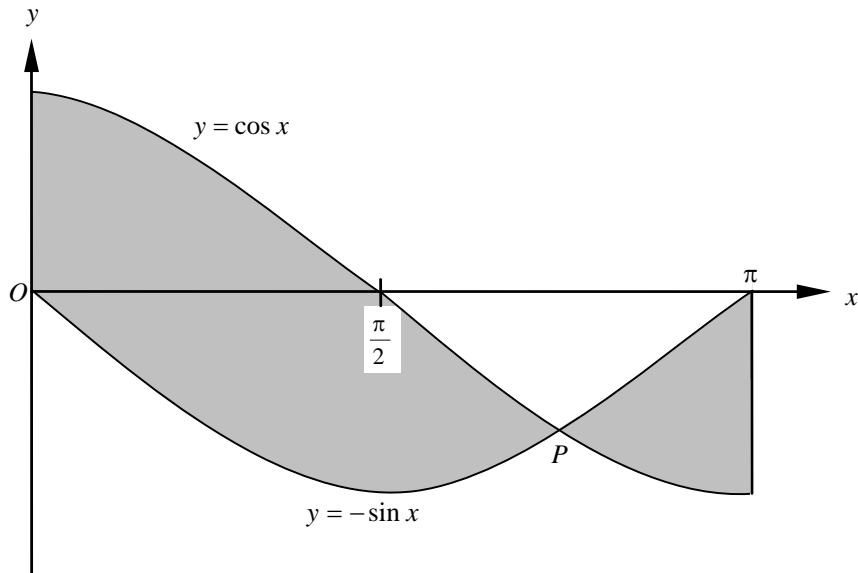


Figure 1

圖 1

- (a) Find the x -coordinate of P .
求 P 的 x 坐標。
- (b) Find the area of the shaded region.
求陰影部份的面積。

(5 marks)

3. Find the general solution of the equation $\sin 2\theta - \sin 3\theta + \sin 4\theta = 0$.
求方程 $\sin 2\theta - \sin 3\theta + \sin 4\theta = 0$ 的通解。

(5 marks)

Please do not write in the margin.

8. Sand falls onto a horizontal table at a rate of $10 \text{ cm}^3 \text{ min}^{-1}$ and form a heap in the shape of a right circular cone with vertical angle 60° as shown in Figure 2.
沙以 $10 \text{ cm}^3 \text{ min}^{-1}$ 之速率降落在水平桌上並形成一正圓錐體形狀，如圖 2 所示，圓錐體的頂角為 60° 。

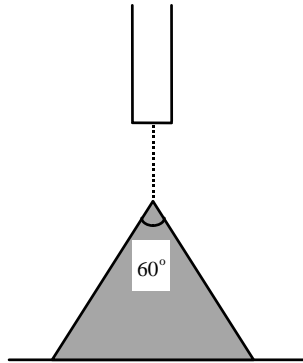


Figure 2

圖 2

Find the rate of increase of the height of the heap with respect to time when the heap is at a height of 15 cm.

求當沙堆的高度為 15 cm 時，高度上升的速率。

(5 marks)

9. $(1 + ax)^n = 1 + px + qx^2 + \text{terms involving higher powers of } x$, where n is a positive integer.

$(1 + ax)^n = 1 + px + qx^2 + x$ 的更高次冪的項，其中 n 為一正整數。

- (a) Express p and q in terms of a and n .

以 a 及 n 表 p 及 q 。

- (b) If $p = 21$ and $q = 189$, find a and n .

若 $p = 21$ 及 $q = 189$ ，求 a 及 n 。

(6 marks)

10. L is a line of slope m , where $m > 2$, which makes an angle θ with the line $2x - y + 13 = 0$ where $\tan \theta = \frac{1}{7}$.

直線 L 的斜率為 m ，其中 $m > 2$ ，且與直線 $2x - y + 13 = 0$ 的夾角為 θ ，其中 $\tan \theta = \frac{1}{7}$ 。

- (a) Find the value of m .

求 m 的值。

- (b) If the distance from $(2, -3)$ to the line L is $2\sqrt{10}$, find the two equations of L .

若從 $(2, -3)$ 至直線 L 的距離為 $2\sqrt{10}$ ，求 L 的兩個方程。

(6 marks)

11. Prove by mathematical induction that

利用數學歸納法證明對所有正整數 n ，

$$2 \times 2 + 3 \times 2^2 + 4 \times 2^3 + \cdots + (n+1) \cdot 2^n = n \cdot 2^{n+1}$$

for all positive integers n .

(6 marks)

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12. The slope of tangent at any point (x, y) of a curve P is given by $\frac{dy}{dx} = 2 \sin \frac{x}{2}$. The line $y = x - \frac{\pi}{3}$ is a tangent to the curve at point A for $0 \leq x \leq \pi$. Find

曲線 P 在任意一點 (x, y) 切線的斜率是 $\frac{dy}{dx} = 2 \sin \frac{x}{2}$ 。直線 $y = x - \frac{\pi}{3}$ 與曲線於點 A 相切，其中 $0 \leq x \leq \pi$ 。求

- (a) the coordinates of A .
 A 的坐標；
- (b) the equation of P .
 P 的方程。

(6 marks)

Please do not write in the margin.

SECTION B (48 marks)

乙部 (48 分)

Answer any **FOUR** questions in this section. Each question carries 12 marks.

本部選答四題，每題 12 分。

13. In Figure 3, $ABCD$ is a parallelogram. E and F are mid-points of AD and DC respectively. G and H are points on AB and BC respectively such that $AG : GB = 1 : 2$ and $CH : HB = 1 : 3$. EH meets FG at a point X such that $GX : XF = t : 1 - t$, where $t > 0$. Let $\vec{DA} = \mathbf{a}$ and $\vec{DC} = \mathbf{b}$.

在圖 3， $ABCD$ 為一平行四邊形。 E 及 F 分別為 AD 及 DC 的中點。 G 及 H 分別為 AB 及 BC 上的點且 $AG : GB = 1 : 2$ 及 $CH : HB = 1 : 3$ 。 EH 與 FG 相交於點 X 且 $GX : XF = t : 1 - t$ ，其中 $t > 0$ 。設 $\vec{DA} = \mathbf{a}$ 及 $\vec{DC} = \mathbf{b}$ 。

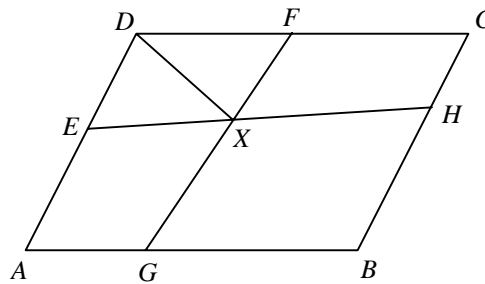


Figure 3

圖 3

- (a) (i) Express \vec{DG} in terms of \mathbf{a} and \mathbf{b} .

以 \mathbf{a} 及 \mathbf{b} 表 \vec{DG} 。

- (ii) Hence, show that

由此，證明

$$\vec{DX} = (1-t)\mathbf{a} + \frac{t+2}{6}\mathbf{b}.$$

(4 marks)

- (b) (i) Express \vec{DH} in terms of \mathbf{a} and \mathbf{b} .

以 \mathbf{a} 及 \mathbf{b} 表 \vec{DH} 。

- (ii) Hence, express \vec{EH} in terms of \mathbf{a} and \mathbf{b} .

由此，以 \mathbf{a} 及 \mathbf{b} 表 \vec{EH} 。

- (iii) Using the fact that E , X and H are collinear, or otherwise, show that $t = \frac{14}{23}$.

利用 E 、 X 及 H 為共線的事實，證明 $t = \frac{14}{23}$ 。

(8 marks)

14. In Figure 4, $CDEF$ and $ABFE$ are two rectangular planes perpendicular to each other. X, H are points on AD and AE respectively such that $XH \perp AE$. Let $\angle CAF = \alpha$, $\angle XBH = \beta$ and $\angle CBF = \phi$.

在圖 4 中， $CDEF$ 及 $ABFE$ 為兩互相垂直之長方形平面。 X, H 為 AD 及 AE 上之點，且 $XH \perp AE$ 。設 $\angle CAF = \alpha$ ， $\angle XBH = \beta$ ， $\angle CBF = \phi$ 。

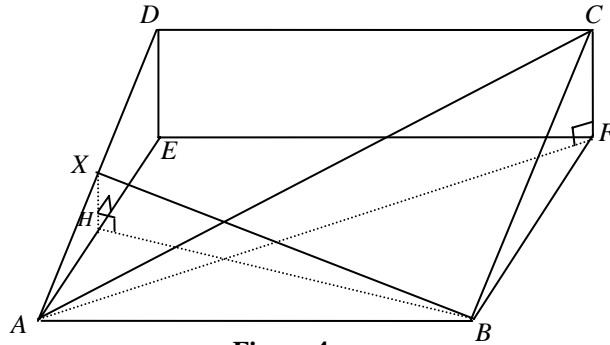


Figure 4
圖 4

- (a) By considering $\triangle ACB$, prove that $\cos \angle ACB = \frac{\sin \alpha}{\sin \phi}$.

透過考慮 $\triangle ACB$ ，證明 $\cos \angle ACB = \frac{\sin \alpha}{\sin \phi}$ 。

(3 marks)

- (b) By considering $\triangle ABX$, express $\cos \angle AXB$ in terms of β and ϕ .

透過考慮 $\triangle ABX$ ，以 β 及 ϕ 表 $\cos \angle AXB$ 。

(2 marks)

- (c) If $AC \perp BX$, find the value of $\sin^2 \alpha + \sin^2 \beta - \sin^2 \phi$.

若 $AC \perp BX$ ，試求 $\sin^2 \alpha + \sin^2 \beta - \sin^2 \phi$ 之值。

A student claims that it is impossible to have $AC \perp BX$ with $\alpha > 45^\circ$ and $\beta > 45^\circ$, explain whether the claim is correct.

某學生指出不能使 $AC \perp BX$ ，且 $\alpha > 45^\circ$ 及 $\beta > 45^\circ$ ，解釋該學生的說法是否正確。

(7 marks)

15. Figure 5 shows a circle $C : (x - 2)^2 + y^2 = 1$ and a straight line $L : y = kx + a$, where k and a are constants.

圖 5 所示為圓 $C : (x - 2)^2 + y^2 = 1$ 及直線 $L : y = kx + a$ ，其中 k 及 a 為常數。

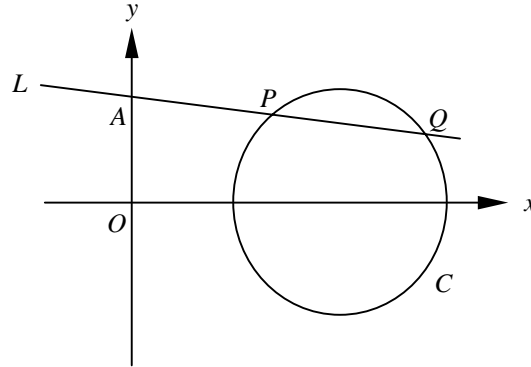


Figure 5

圖 5

L cuts the y -axis at the point A and the circle C at two distinct points $P(p, p')$ and $Q(q, q')$.

$R(x, y)$ is a point on the line segment PQ such that $PR : RQ = AP : AQ$.

L 與 y 軸相交於 A 而與圓 C 相交於 $P(p, p')$ 、 $Q(q, q')$ 。 $R(x, y)$ 為線段 PQ 上的一點使到 $PR : RQ = AP : AQ$ 。

- (a) Express $AP : AQ$ in terms of p and q .

以 p 及 q 表 $AP : AQ$ 。

Hence, express the x -coordinate of R in terms of p and q .

由此，以 p 及 q 表 R 的 x 坐標。

(3 marks)

- (b) Find the coordinates of R in terms of a and k . Hence, find the equation of the locus of R in terms of a as k varies.

以 a 及 k 表 R 的坐標。由此，以 a 表當 k 變化時 R 的軌跡方程。

(7 marks)

- (c) Given that whatever the value of a , the locus obtained in (b) always passes through a fixed point. Write down the coordinates of the fixed point.

已知無論 a 的值，(b) 中的軌跡一定通過某一定點。寫出該定點的坐標。

(2 marks)

16. Figure 6 shows a sector containing an angle θ (where $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$) is cut off from a circle of radius r . The remaining part is then folded to form a circular cone.

圖 6 所示為某扇形從一半徑為 r 的圓中被剪出，而扇形的角為 θ (其中 $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$)。而剩餘的部分被摺成一圓錐體。

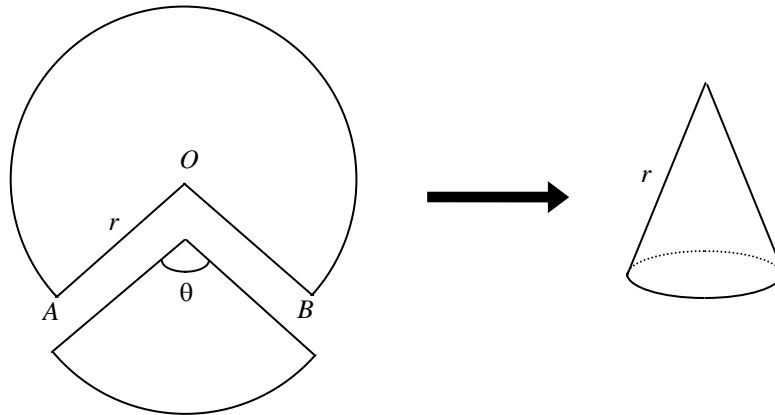


Figure 6
圖 6

- (a) (i) Show that the total surface area (including the base) of the cone is given by
證明圓錐體之總表面面積 (包括底部) 為

$$S = \frac{r^2}{4\pi} (\theta^2 - 6\pi\theta + 8\pi^2).$$

- (ii) Show that S decreases as θ increases.

當 θ 增加，證明 S 為遞減的。

Hence, write down the value of θ such that S attains its maximum value.

由此，寫出當 S 達到其極大值時 θ 的值。

(6 marks)

- (b) It is given that the volume of the cone is given by
已知圓錐的體積為

$$V = \frac{r^3}{24\pi^2} (2\pi - \theta)^2 \sqrt{4\pi\theta - \theta^2}.$$

- (i) Find the value of θ such that V is maximum.

若 V 為極大值，求 θ 的值

(Testing maximum/minimum is not required.)

(考生不用寫出驗算極大/極小的步驟。)

- (ii) Does S attain a maximum when V attains its maximum? Explain your answer.

當 V 為極大值，問 S 是否亦為極大值? 試解釋。

(6 marks)

17. Figure 7 shows a longitudinal section of a bowl generated by revolving the lower half of the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about the y -axis. The bowl contains water to a depth of h units, where $0 \leq h \leq 3$.

圖 7 所示為一由曲線 $\frac{x^2}{16} + \frac{y^2}{9} = 1$ 的下半部繞 y 軸旋轉而形成的碗的縱切面。該碗盛水至 h 單位深，其中 $0 \leq h \leq 3$ 。

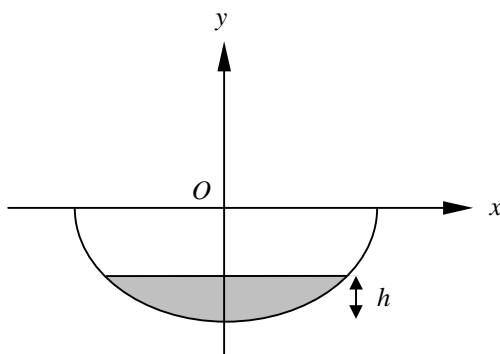


Figure 7

圖 7

- (a) Show that the volume of water in the bowl is $\frac{16}{27}\pi h^2(9-h)$ cubic units.

證明碗中水的體積為 $\frac{16}{27}\pi h^2(9-h)$ 立方單位。

(4 marks)

- (b) Find the capacity of the bowl.
求碗的容積。

(2 marks)

- (c) Given that $\frac{23}{27}$ of the bowl is empty initially.

已知開始時 $\frac{23}{27}$ 的碗是空的。

- (i) Find the depth of water in the bowl.
求碗中水的深度。

- (ii) Now more water is added at a rate of $\frac{\pi}{9}(k+t)$ cubic units per hour at time t hour.

After 12 hours, the bowl is full of water. Find the value of the constant k .

於 t 時，水以每小時 $\frac{\pi}{9}(k+t)$ 立方單位的速率倒入碗中。經過 12 小時，碗便盛滿了水。求常數 k 的值。

(6 marks)

END OF PAPER
全卷完